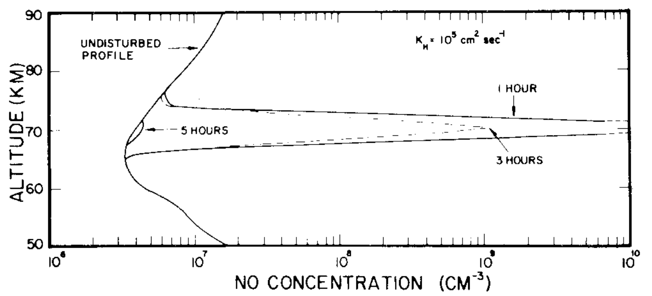

Fig. 4 NO perturbation for horizontal diffusion coefficient = 10^5 cm² sec^{-1}.

done by appending to the numerical solution an analytic solution of a diffusion equation in the horizontal. The horizontal eddy or molecular diffusion coefficient appears as a parameter in this diffusion equation. This allows us to watch the material leak out the sides of a box perpendicular to the trajectory of the orbiter. The disturbance has been initially spread into a box 1 km wide in both the horizontal and vertical. That point in time is labeled $t = 0$ and the solution is begun from there considering vertical diffusion plus horizontal depletion from the 1 km box. Thus the perturbed concentrations are averaged over a box $(1 \text{ km})^2$ perpendicular to the trajectory.

Figure 2 shows the undisturbed background profile of NO with several perturbed profiles shown also. The times shown are 1, 3, and 8 hr after the time when the perturbation had spread to $\approx (1 \text{ km})^2$ area around the initial wake line source. The horizontal diffusion coefficient was taken to be 10^3 at 50 km up to 10^5 at 90 km varying exponentially. Relaxation to ambient took place after approximately 110 hours. Figure 3 shows NO^+ vs time for the same case. The disturbance is again long lasting and, while easily detectable, is not large in absolute value.

Figures 4 and 5 show similar calculations in which the horizontal diffusion coefficients were taken to be 10^5 cm² sec^{-1} independent of altitude. In this case relaxation to ambient took place in ~ 5 hr. If the horizontal diffusion coefficient is larger yet, there will be a corresponding decrease in the lifetime of the disturbance.

III. Conclusions and Summary

We have modeled the localized effect of a single shuttle orbiter re-entry on mesospheric odd nitrogen. The perturbations of the odd nitrogen species last for a time of the order of hours depending critically upon the value used for the horizontal diffusion coefficient. The processes included in the calculation are the perturbation of NO, photochemical reactions, and horizontal and vertical eddy diffusion. Effects which have not been modeled but may be important are perturbations in N and O, wind shear, diurnal variations, and the influence of heat shield ablation products or water vapor. We have not considered specifically the problem of buildup from repeated orbiter re-entries at the same place. However, the results indicate that even in the worst case the disturbance lifetime is less than 5 days. This combined with the fact that winds will move the disturbance fairly rapidly indicates that there is little possibility of buildup for predicted launch frequencies.

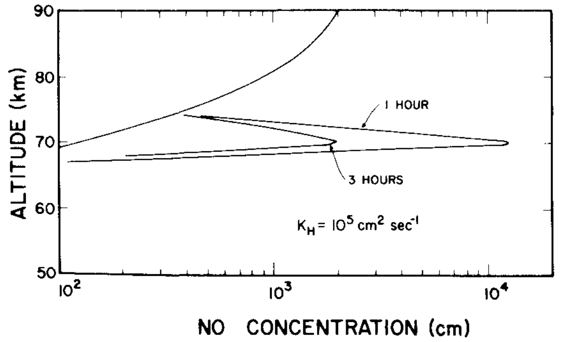

Fig. 5 NO^+ perturbation for horizontal diffusion coefficient = 10^5 cm² sec^{-1}.

References

[1] Park, C., "Estimates of Nitric Oxide Production for Lifting Spacecraft Re-entry," Report TM-X-62-052, 1972, NASA.

[2] Strobel, D. F., "Odd Nitrogen in the Mesosphere," *Journal of Geophysical Research*, Vol. 76, 1971b, p. 8384.

[3] Strobel, D. F., "Diurnal Variation of Nitric Oxide in the Upper Atmosphere," *Journal of Geophysical Research*, Vol. 76, 1971, p. 2441.

Correlation of Peak Heating in the Reattachment Region of Separated Flows

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1. Introduction

VERY high local values of the heat-transfer rate have been measured in the reattachment region of separated boundary layers in high speed flows. Bushnell and Weinstein[1] have derived successful correlation parameters for both laminar and turbulent regimes. The present Note gives some theoretical support to this empirical correlation in the laminar case and shows how the results of some preliminary calculations could lead to a substantial improvement in the correlation technique.

2. Background

The derivation of Bushnell and Weinstein's parameters is based on the assumption that at reattachment a sublayer is formed, the behavior of which is governed by the properties of the incoming boundary layer being compressed above it. The idealized flowfield is shown in Fig. 1.

The authors argue that flat plate-type relationships can be applied in the reattachment region with the peak Stanton number and the corresponding Reynolds number evaluated under local conditions. These correlation parameters are thus defined as the following:

$$(h_{max}/\rho_\omega u_3 c_p) \alpha (\rho_\omega u_3 x_p/\mu_\omega)^{-0.5}$$

where h_{max} = maximum heat-transfer coefficient; ρ_ω = density (note $\rho_\omega \alpha P_3/T_\omega$); μ_ω = viscosity evaluated at wall temperature T_ω; c_p = specific heat at constant pressure; u_3 = inviscid velocity in region 3 (see Fig. 1); and x_p = distance between reattachment and peak heating.

Bushnell and Weinstein are able to correlate the results of six different experiments which involve flap induced laminar boundary-layer separations with freestream Mach numbers

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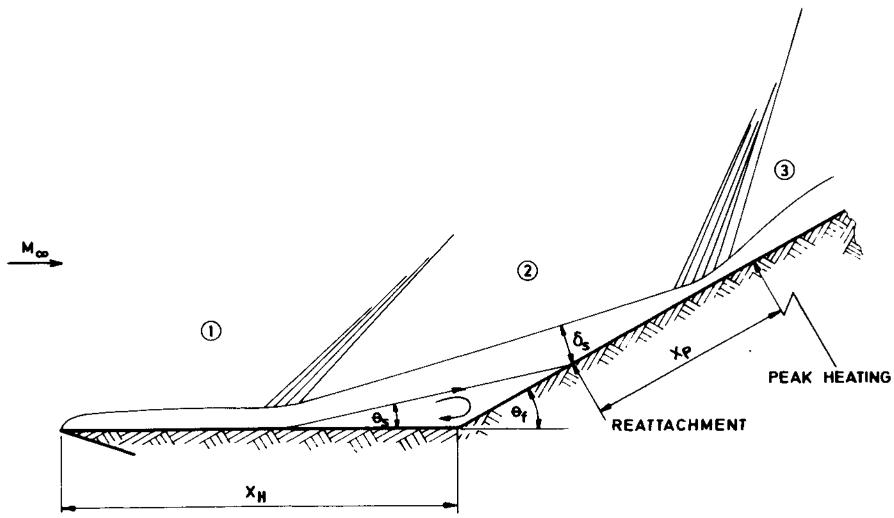

Fig. 1 Idealized flowfield.

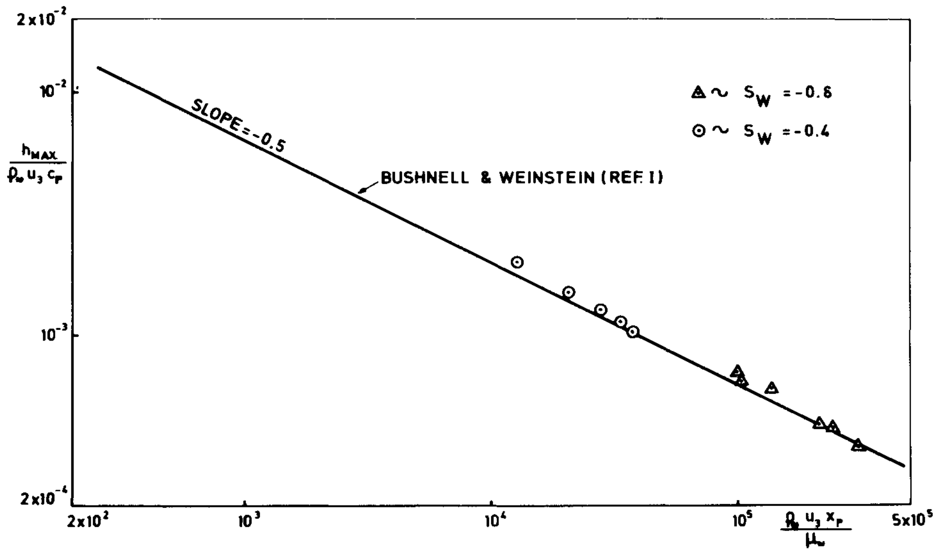

Fig. 2 Peak Stanton number correlation.

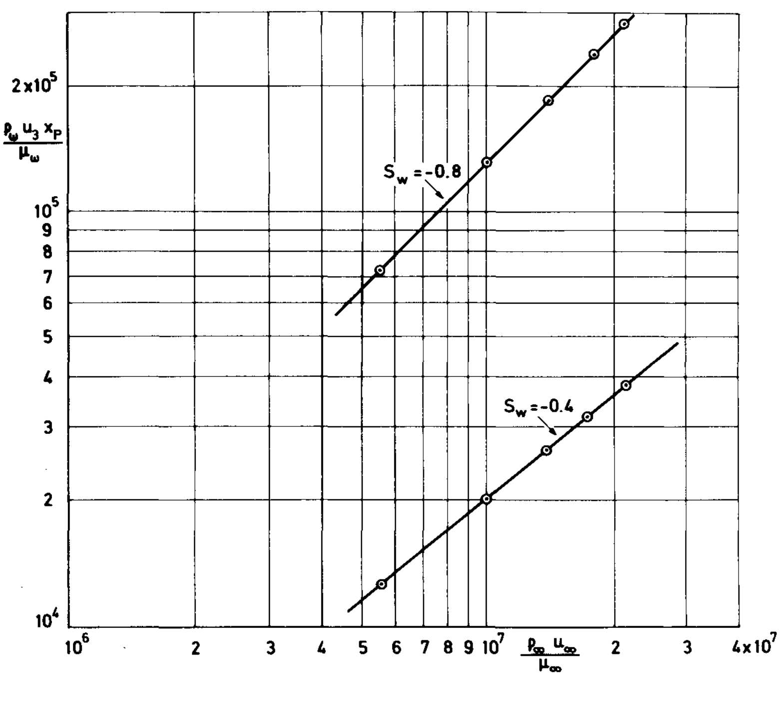

Fig. 3 Correlation between reattachment and free-stream Reynolds numbers.

between 10 and 20 and with intense wall cooling. The correlation line obtained is of the form

$$St_{\max} = 0.2(Re_{x_p})^{-0.5}$$

The major source of difficulty when attempting to apply this correlation in a practical case lies in the evaluation of x_p. Bushnell and Weinstein make the assumption that the separation geometry is known and propose that if the peak heating occurs near the peak pressure, then from Fig. 1 the following relation can be applied:

$$x_p = [\delta_s/\sin(\theta_f - \theta_s)]$$

Even with schlieren photographs of the flow, small errors in $(\theta_f - \theta_s)$ are bound to occur and when $\sin(\theta_f - \theta_s)$ is a small quantity, large errors can arise in the estimation of x_p. Therefore, in general engineering applications the correlation is difficult to use.

This Note describes some preliminary results of a theoretical investigation which basically support Bushnell and Weinstein's ideas but which indicate a more direct method of predicting the peak heat-transfer coefficients. The theory used is Gautier and Ginoux[2] improved version of Klineberg's[3] integral method for viscous-inviscid interactions in moderate supersonic flows.

3. Results and Discussion

The calculations were carried out for a freestream Mach number of 6.0 and a Reynolds number range of 5×10^6 to 3×10^7/m. The flap angle θ_f was 7.5° and the hinge line was 40 mm from the leading edge. Two values of total enthalpy function s_w were used, −0.4 and −0.8, where $s_w = (T_w/T_{0_\infty}) - 1$.

Peak Stanton numbers and reattachment Reynolds numbers were calculated, with the distance x_p being given directly in the computed results. Figure 2 shows a comparison between the experimental correlation line obtained by Bushnell and Weinstein and the results of the present calculations. They are seen to be in excellent agreement. The determination of the initial conditions for the interaction calculations demands that $[M_\infty{}^3(c)^{1/2}/(Re_x)^{1/2}]$ is a small quantity where M_∞ is the freestream Mach number, c is the Chapman constant, and Re_x is the Reynolds number based on the distance from the leading edge. This fact necessitates the use of high freestream Reynolds numbers so that the computed results tend to be grouped to the right of Fig. 2.

The encouraging results of Fig. 2 prompted a more detailed study of the theoretical correlation parameters in the hope that the correlation technique itself might be improved. This exercise produced the useful though not totally unexpected result that for a fixed value of s_w the reattachment Reynolds number Re_{x_p} was only a function of the freestream Reynolds number Re_∞. These relationships are

$$s_w = -0.4 \qquad Re_{x_p} = 0.27(Re_\infty)^{0.82}$$

$$s_w = -0.8 \qquad Re_{x_p} = 0.013 Re_\infty$$

They are plotted in Fig. 3.

If the generality of these equations could be increased to cover various freestream conditions and flat plate/wedge geometries, then in practical cases the reattachment Reynolds number could be calculated directly, obviating the need to estimate x_p.

References

[1] Bushnell, D. M. and Weinstein, L. M., "Correlation of Peak Heating for Reattachment of Separated Flows," *Journal of Spacecraft and Rockets*, Vol. 5, No. 9, Sept. 1968, pp. 1111–1112.

[2] Gautier, B. and Ginoux, J. J., "Improved Computer Program for Calculation of Viscous-Inviscid Interactions," TN 82, March 1973, von Kármán Institute, Rhode-St. Genèse, Belgium; see also *AIAA Journal*, Vol. 11, No. 9, Sept. 1973, pp. 1323–1326.

[3] Klineberg, J. M., "Theory of Laminar Viscous-Inviscid Interactions in Supersonic Flow," PhD thesis, OSR 68-1569, 1968, Graduate Aeronautical Lab., California Institute of Technology, Pasadena, Calif.

Applications of Bolotin's Method to Vibrations of Plates

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VARIOUS techniques have been brought to bear on the problem of determining natural frequencies and modes of free transverse vibration of thin elastic plates. Generally, these methods (finite difference, finite element, Rayleigh-Ritz, Fourier series) do not provide accurate numerical results for other than the lowest modes without a substantial, and perhaps prohibitive, investment in digital computation. On the other hand an asymptotic technique developed by Bolotin[1–4] over a decade ago for eigenvalue problems in rectangular regions has the property of greater accuracy for higher modes than for lower modes. Not only does Bolotin's method complement the usual approximate methods of analysis for linear eigenvalue problems, but Bolotin and his co-workers found it to yield, with remarkable accuracy, fundamental natural frequencies for isotropic, and in one instance orthotropic, rectangular plates with supported edges. The purposes of this Note are to call attention to Bolotin's method which has not been widely discussed in the Western literature, to compare results obtained from it with recently published and apparently accurate natural frequencies of orthotropic plates, and to discuss its applicability to problems in which a plate is not supported on some part of its boundary.

We restrict our attention to the case of a uniform rectangular plate lying in the xy-plane ($0 < x < a, 0 < y < b$) such that the axes of symmetry of the orthotropic material are parallel to the edges of the plate. It will be clear from the analysis that follows that these restrictions, along with the requirement that boundary conditions be uniform along any edge, are necessary to the applicability of the asymptotic method. For free vibration the transverse displacement $w(x, y) \sin \omega t$ is governed by[5]

$$D_x(\partial^4 w/\partial x^4) + 2H(\partial^4 w/\partial x^2 \partial y^2) + D_y(\partial^4 w/\partial y^4) - \rho\omega^2 w = 0$$

where D_x, H, D_y are flexural rigidities and ρ is the mass per unit area of the plate.

The essence of Bolotin's technique is the assumption that an eigenfunction can be represented adequately by $w = \sin k_x(x - x_o) \sin k_y(y - y_o)$ except near the edges, where the form of an eigenfunction is determined by boundary conditions. This "interior" solution yields a formula for the natural frequency, ω, in terms of the wave numbers k_x and k_y

$$\omega^2 = (1/\rho)[k_x{}^4 D_x + 2k_x{}^2 k_y{}^2 H + k_y{}^4 D_y]$$

We now seek solutions to the governing equation to describe an eigenfunction near edges of the plate. To represent an eigenfunction near $x = 0$ or $x = a$, we let

$$w(x, y) = \phi(x) \sin k_y(y - y_o)$$

hence ϕ is governed by

$$\phi'''' - (2Hk_y{}^2/D_x)\phi'' - k_x{}^2[k_x{}^2 + (2Hk_y{}^2/D_x)]\phi = 0$$

and with $\phi = Ae^{\gamma x}$

$$\gamma^2 = -k_x{}^2$$

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